

SM3 8.1 Exponential and Log Equations

Solve the following equations

1. $3^{n-2} = 27$

$$\begin{aligned}\log_3 3^{n-2} &= \log_3 27 \\ n - 2 &= 3 \\ n &= 5\end{aligned}$$

2. $2^{3x+5} = 128$

$$\begin{aligned}\log_2 2^{3x+5} &= \log_2 128 \\ 3x + 5 &= 7 \\ 3x &= 2 \\ x &= \frac{2}{3}\end{aligned}$$

3. $5^{n-3} = \frac{1}{25}$

$$\begin{aligned}\log_5 5^{n-3} &= \log_5 \frac{1}{25} \\ n - 3 &= -2 \\ n &= 1\end{aligned}$$

4. $10^{x-1} = 100^{2x-3}$

$$\begin{aligned}10^{x-1} &= (10^2)^{2x-3} \\ 10^{x-1} &= 10^{4x-6} \\ \log 10^{x-1} &= \log 10^{4x-6} \\ x - 1 &= 4x - 6 \\ -3x &= -5 \\ x &= \frac{5}{3}\end{aligned}$$

5. $\log_9 x = 2$

$$\begin{aligned}9^{\log_9 x} &= 9^2 \\ x &= 81\end{aligned}$$

6. $\log_{25} n = \frac{3}{2}$

$$\begin{aligned}25^{\log_{25} n} &= 25^{\frac{3}{2}} \\ n &= 5^3 \\ n &= 125\end{aligned}$$

7. $\log_{\frac{1}{7}} x = -1$

$$\begin{aligned}\frac{1}{7}^{\log_{\frac{1}{7}} x} &= \frac{1}{7}^{-1} \\ \frac{1}{7} &= \frac{1}{7} \\ x &= 7\end{aligned}$$

8. $\log(x^2 + 1) = 1$

$$\begin{aligned}10^{\log(x^2+1)} &= 10^1 \\ x^2 + 1 &= 10 \\ x^2 &= 9 \\ x &= \pm 3\end{aligned}$$

9. $\log_b 64 = 3$

$$\begin{aligned}b^{\log_b 64} &= b^3 \\ 64 &= b^3 \\ 4 &= b\end{aligned}$$

10. $\log_5 5^{6n+1} = 13$

$$\begin{aligned}6n + 1 &= 13 \\ 6n &= 12 \\ n &= 2\end{aligned}$$

11. $\log_5 x = \frac{1}{2}$

$$\begin{aligned}5^{\log_5 x} &= 5^{\frac{1}{2}} \\ x &= \sqrt{5}\end{aligned}$$

12. $\log_b 121 = 2$

$$\begin{aligned}b^{\log_b 121} &= b^2 \\ 121 &= b^2 \\ \pm 11 &= b\end{aligned}$$

13. $\log_6(2x - 3) = \log_6(x + 2)$

$$\begin{aligned}6^{\log_6(2x-3)} &= 6^{\log_6(x+2)} \\ 2x - 3 &= x + 2 \\ x &= 5\end{aligned}$$

14. $\log_7(x^2 + 36) = \log_7 100$

$$\begin{aligned}x^2 + 36 &= 100 \\ x^2 &= 64 \\ x &= \pm 8\end{aligned}$$

15. $\log_3 5 + \log_3 x = \log_3 10$

$$\begin{aligned}\log_3 5x &= \log_3 10 \\ 3^{\log_3 5x} &= 3^{\log_3 10} \\ 5x &= 10 \\ x &= 2\end{aligned}$$

16. $\log_4 a + \log_4 9 = \log_4 27$

$$\begin{aligned}\log_4 9a &= \log_4 27 \\ 4^{\log_4 9a} &= 4^{\log_4 27} \\ 9a &= 27 \\ a &= 3\end{aligned}$$

$$17. \log 16 - \log 2t = \log 2$$

$$\begin{aligned} \log \frac{8}{t} &= \log 2 \\ 10^{\log \frac{8}{t}} &= 10^{\log 2} \\ \frac{8}{t} &= 2 \\ \frac{t}{8} &= \frac{1}{2} \\ t &= 4 \end{aligned}$$

$$18. \log_7 24 - \log_7(y+5) = \log_7 8$$

$$\begin{aligned} \log_7 \frac{24}{y+5} &= \log_7 8 \\ 7^{\log_7 \frac{24}{y+5}} &= 7^{\log_7 8} \\ \frac{24}{y+5} &= 8 \\ \frac{y+5}{24} &= \frac{1}{8} \\ y+5 &= 3 \\ y &= -2 \end{aligned}$$

$$19. \log_2 n = \frac{1}{4} \log_2 16 + \frac{1}{2} \log_2 49$$

$$\begin{aligned} \log_2 n &= \log_2 16^{\frac{1}{4}} + \log_2 49^{\frac{1}{2}} \\ \log_2 n &= \log_2 2 + \log_2 7 \\ \log_2 n &= \log_2 14 \\ 2^{\log_2 n} &= 2^{\log_2 14} \\ n &= 14 \end{aligned}$$

$$20. 2 \log 6 - \frac{1}{3} \log 27 = \log x$$

$$\begin{aligned} \log 6^2 - \log 27^{\frac{1}{3}} &= \log x \\ \log 36 - \log 3 &= \log x \\ \log 12 &= \log x \\ 10^{\log 12} &= 10^{\log x} \\ 12 &= x \end{aligned}$$

$$21. \log z + \log(z+3) = 1$$

$$\begin{aligned} \log(z^2 + 3z) &= 1 \\ 10^{\log(z^2 + 3z)} &= 10^1 \\ z^2 + 3z &= 10 \\ z^2 + 3z - 10 &= 0 \\ (z+5)(z-2) &= 0 \\ z &= \{-5, 2\} \end{aligned}$$

As -5 fails to check, we eliminate it.

$$z = 2$$

$$22. \log_6(a^2 + 2) + \log_6 2 = 2$$

$$\begin{aligned} \log_6(2a^2 + 4) &= 2 \\ 6^{\log_6(2a^2 + 4)} &= 6^2 \\ 2a^2 + 4 &= 36 \\ 2a^2 &= 32 \\ a^2 &= 16 \\ a &= \pm 4 \end{aligned}$$

$$23. \log_2(12b - 21) - \log_2(b^2 - 3) = 2$$

$$\begin{aligned} \log_2 \left(\frac{12b - 21}{b^2 - 3} \right) &= 2 \\ 2^{\log_2 \left(\frac{12b - 21}{b^2 - 3} \right)} &= 2^2 \\ \frac{12b - 21}{b^2 - 3} &= 4 \\ 12b - 21 &= 4b^2 - 12 \\ 4b^2 - 12b + 9 &= 0 \\ (2b - 3)^2 &= 0 \\ 2b - 3 &= 0 \\ 2b &= 3 \\ b &= \frac{3}{2} \end{aligned}$$

As $\frac{3}{2}$ fails to check, we eliminate it.

$$b = \emptyset$$

$$24. \log_2(y+2) - \log_2(y-2) = 1$$

$$\begin{aligned} \log_2 \left(\frac{y+2}{y-2} \right) &= 1 \\ 2^{\log_2 \left(\frac{y+2}{y-2} \right)} &= 2^1 \\ \frac{y+2}{y-2} &= 2 \\ y+2 &= 2y-4 \\ -y &= -6 \\ y &= 6 \end{aligned}$$

$$25. \log_3 0.1 + 2 \log_3 x = \log_3 2 + \log_3 5$$

$$\log_3 0.1 + \log_3 x^2 = \log_3 2 + \log_3 5$$

$$\log_3 0.1x^2 = \log_3 10$$

$$3^{\log_3 0.1x^2} = 3^{\log_3 10}$$

$$0.1x^2 = 10$$

$$x^2 = 100$$

$$x = \pm 10$$

As -10 fails to check, we reject it.

$$x = 10$$

$$26. \log_5 64 - \log_5 \frac{8}{3} + \log_5 2 = \log_5 4p$$

$$\log_5 128 - \log_5 \frac{8}{3} = \log_5 4p$$

$$\log_5 \frac{128}{8/3} = \log_5 4p$$

$$\log_5 128 \cdot \frac{3}{8} = \log_5 4p$$

$$\log_5 16 \cdot 3 = \log_5 4p$$

$$\log_5 48 = \log_5 4p$$

$$5^{\log_5 48} = 5^{\log_5 4p}$$

$$48 = 4p$$

$$12 = p$$

$$27. 2e^x - 1 = 0$$

$$2e^x = 1$$

$$e^x = \frac{1}{2}$$

$$x = \ln\left(\frac{1}{2}\right)$$

$$28. -3e^{4x} + 11 = 2$$

$$-3e^{4x} = -9$$

$$e^{4x} = 3$$

$$4x = \ln 3$$

$$x = \frac{\ln 3}{4}$$

$$29. \ln 2x = 4$$

$$2x = e^4$$

$$x = \frac{e^4}{2}$$

$$30. \ln 3x = 5$$

$$3x = e^5$$

$$x = \frac{e^5}{3}$$

$$31. \ln(x + 1) = 1$$

$$x + 1 = e$$

$$x = e - 1$$

$$32. \ln(x - 7) = 2$$

$$x - 7 = e^2$$

$$x = e^2 + 7$$

$$33. \ln x + \ln 3x = 12$$

$$\ln 3x^2 = 12$$

$$3x^2 = e^{12}$$

$$x^2 = \frac{e^{12}}{3}$$

$$x = \pm \frac{e^6}{\sqrt{3}} = \pm \frac{e^6 \sqrt{3}}{3}$$

The negative result fails to check.

$$x = \frac{e^6 \sqrt{3}}{3}$$

$$34. \ln 4x + \ln x = 9$$

$$\ln 4x^2 = 9$$

$$4x^2 = e^9$$

$$x^2 = \frac{e^9}{4}$$

$$x = \pm \frac{e^{4.5}}{2}$$

The negative result fails to check.

$$x = \frac{e^{4.5}}{2}$$

$$35. \ln(x^2 + 12) = \ln x + \ln 8$$

$$\ln(x^2 + 12) = \ln 8x$$

$$x^2 + 12 = 8x$$

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = \{2, 6\}$$

$$36. \ln x + \ln(x + 4) = \ln 5$$

$$\ln(x^2 + 4x) = \ln 5$$

$$x^2 + 4x = 5$$

$$x^2 + 4x - 5 = 0$$

$$(x + 5)(x - 1) = 0$$

$$x = \{-5, 1\}$$

-5 fails to check.

$$x = 1$$

$$37. \quad e^{\ln x} = 4$$
$$x = 4$$

$$38. \quad 200e^{-4x} = 15$$
$$e^{-4x} = \frac{3}{40}$$
$$-4x = \ln\left(\frac{3}{40}\right)$$
$$x = -\frac{1}{4}\ln\left(\frac{3}{40}\right)$$

$$39. \quad \ln x^2 = 10$$
$$x^2 = e^{10}$$
$$x = \pm e^5$$

$$40. \quad 9 - 2e^x = 7$$
$$-2e^x = -2$$
$$e^x = 1$$
$$x = 0$$

$$41. \quad \ln \sqrt{x+2} = 1$$
$$\sqrt{x+2} = e$$
$$x+2 = e^2$$
$$x = e^2 - 2$$

$$42. \quad \ln(x-2)^2 = 12$$
$$(x-2)^2 = e^{12}$$
$$x-2 = \pm e^6$$
$$x = 2 \pm e^6$$