SM3 8.1 Exponential and Log Equations

Solve the following equations

1. $3^{n-2} = 27$

$\log_3 3^{n-2} = \log_3 27$

$n - 2 = 3$

$n = 5$

2. $2^{3x+5} = 128$

$\log_2 2^{3x+5} = \log_2 128$

$3x + 5 = 7$

$3x = 2$

$x = \frac{2}{3}$

3. $5^{n-3} = \frac{1}{25}$

$\log_5 5^{n-3} = \log_5 \frac{1}{25}$

$n - 3 = -2$

$n = 1$

4. $10^{x-1} = 100^{2x-3}$

$10^{x-1} = (10^2)^{2x-3}$

$10^{x-1} = 10^{4x-6}$

$\log 10^{x-1} = \log 10^{4x-6}$

$x - 1 = 4x - 6$

$-3x = -5$

$x = \frac{5}{3}$

5. $\log_9 x = 2$

$9^{\log_9 x} = 9^2$

$x = 81$

6. $\log_{25} n = \frac{3}{2}$

$25^{\log_{25} n} = 25^{\frac{3}{2}}$

$n = 5^3$

$n = 125$

7. $\log_{\frac{1}{7}} x = -1$

$\frac{1}{7}^{\log_{\frac{1}{7}} x} = \frac{1}{7}^{-1}$

$x = 7$

8. $\log(x^2 + 1) = 1$

$10^{\log(x^2+1)} = 10^1$

$x^2 + 1 = 10$

$x^2 = 9$

$x = \pm 3$

9. $\log_b 64 = 3$

$b^{\log_b 64} = b^3$

$64 = b^3$

$4 = b$

10. $\log_5 5^{6n+1} = 13$

$6n + 1 = 13$

$6n = 12$

$n = 2$

11. $\log_5 x = \frac{1}{2}$

$5^{\log_5 x} = 5^{\frac{1}{2}}$

$x = \sqrt{5}$

12. $\log_b 121 = 2$

$b^{\log_b 121} = b^2$

$121 = b^2$

$\pm 11 = b$

13. $\log_6(2x - 3) = \log_6(x + 2)$

$6^{\log_6(2x-3)} = 6^{\log_6(x+2)}$

$2x - 3 = x + 2$

$x = 5$

14. $\log_7(x^2 + 36) = \log_7 100$

$x^2 + 36 = 100$

$x^2 = 64$

$x = \pm 8$

15. $\log_3 5 + \log_3 x = \log_3 10$

$\log_3 5x = \log_3 10$

$3^{\log_3 5x} = 3^{\log_3 10}$

$5x = 10$

$x = 2$

16. $\log_4 a + \log_4 9 = \log_4 27$

$\log_4 9a = \log_4 27$

$4^{\log_4 9a} = 4^{\log_4 27}$

$9a = 27$

$a = 3$

17. $\log 16 - \log 2t = \log 2$

$$\log \frac{8}{t} = \log 2$$
$$10^{\log \frac{8}{t}} = 10^{\log 2}$$
$$\frac{8}{t} = 2$$
$$\frac{t}{8} = \frac{1}{2}$$
$$t = 4$$

18. $\log_7 24 - \log_7(y+5) = \log_7 8$

$$\log_7 \frac{24}{y+5} = \log_7 8$$
$$7^{\log_7 \frac{24}{y+5}} = 7^{\log_7 8}$$
$$\frac{24}{y+5} = 8$$
$$\frac{y+5}{24} = \frac{1}{8}$$
$$y + 5 = 3$$
$$y = -2$$

19. $\log_2 n = \frac{1}{4}\log_2 16 + \frac{1}{2}\log_2 49$

$$\log_2 n = \log_2 16^{\frac{1}{4}} + \log_2 49^{\frac{1}{2}}$$
$$\log_2 n = \log_2 2 + \log_2 7$$
$$\log_2 n = \log_2 14$$
$$2^{\log_2 n} = 2^{\log_2 14}$$
$$n = 14$$

20. $2\log 6 - \frac{1}{3}\log 27 = \log x$

$$\log 6^2 - \log 27^{\frac{1}{3}} = \log x$$
$$\log 36 - \log 3 = \log x$$
$$\log 12 = \log x$$
$$10^{\log 12} = 10^{\log x}$$
$$12 = x$$

21. $\log z + \log(z+3) = 1$

$$\log(z^2 + 3z) = 1$$
$$10^{\log(z^2+3z)} = 10^1$$
$$z^2 + 3z = 10$$
$$z^2 + 3z - 10 = 0$$
$$(z+5)(z-2) = 0$$
$$z = \{-5, 2\}$$

As -5 fails to check, we eliminate it.

$$z = 2$$

22. $\log_6(a^2+2) + \log_6 2 = 2$

$$\log_6(2a^2 + 4) = 2$$
$$6^{\log_6(2a^2+4)} = 6^2$$
$$2a^2 + 4 = 36$$
$$2a^2 = 32$$
$$a^2 = 16$$
$$a = \pm 4$$

23. $\log_2(12b-21) - \log_2(b^2-3) = 2$

$$\log_2\left(\frac{12b-21}{b^2-3}\right) = 2$$
$$2^{\log_2\left(\frac{12b-21}{b^2-3}\right)} = 2^2$$
$$\frac{12b-21}{b^2-3} = 4$$
$$12b - 21 = 4b^2 - 12$$
$$4b^2 - 12b + 9 = 0$$
$$(2b-3)^2 = 0$$
$$2b - 3 = 0$$
$$2b = 3$$
$$b = \frac{3}{2}$$

As $\frac{3}{2}$ fails to check, we eliminate it.

$$b = \emptyset$$

24. $\log_2(y+2) - \log_2(y-2) = 1$

$$\log_2\left(\frac{y+2}{y-2}\right) = 1$$
$$2^{\log_2\left(\frac{y+2}{y-2}\right)} = 2^1$$
$$\frac{y+2}{y-2} = 2$$
$$y + 2 = 2y - 4$$
$$-y = -6$$
$$y = 6$$

25. $\log_3 0.1 + 2\log_3 x = \log_3 2 + \log_3 5$

$\log_3 0.1 + \log_3 x^2 = \log_3 2 + \log_3 5$

$\log_3 0.1x^2 = \log_3 10$

$3^{\log_3 0.1x^2} = 3^{\log_3 10}$

$0.1x^2 = 10$

$x^2 = 100$

$x = \pm 10$

As -10 fails to check, we reject it.

$x = 10$

26. $\log_5 64 - \log_5 \frac{8}{3} + \log_5 2 = \log_5 4p$

$\log_5 128 - \log_5 \frac{8}{3} = \log_5 4p$

$\log_5 \frac{128}{8/3} = \log_5 4p$

$\log_5 128 \cdot \frac{3}{8} = \log_5 4p$

$\log_5 16 \cdot 3 = \log_5 4p$

$\log_5 48 = \log_5 4p$

$5^{\log_5 48} = 5^{\log_5 4p}$

$48 = 4p$

$12 = p$

27. $2e^x - 1 = 0$

$2e^x = 1$

$e^x = \frac{1}{2}$

$x = \ln\left(\frac{1}{2}\right)$

28. $-3e^{4x} + 11 = 2$

$-3e^{4x} = -9$

$e^{4x} = 3$

$4x = \ln 3$

$x = \frac{\ln 3}{4}$

29. $\ln 2x = 4$

$2x = e^4$

$x = \frac{e^4}{2}$

30. $\ln 3x = 5$

$3x = e^5$

$x = \frac{e^5}{3}$

31. $\ln(x+1) = 1$

$x + 1 = e$

$x = e - 1$

32. $\ln(x-7) = 2$

$x - 7 = e^2$

$x = e^2 + 7$

33. $\ln x + \ln 3x = 12$

$\ln 3x^2 = 12$

$3x^2 = e^{12}$

$x^2 = \frac{e^{12}}{3}$

$x = \pm\frac{e^6}{\sqrt{3}} = \pm\frac{e^6\sqrt{3}}{3}$

The negative result fails to check.

$x = \frac{e^6\sqrt{3}}{3}$

34. $\ln 4x + \ln x = 9$

$\ln 4x^2 = 9$

$4x^2 = e^9$

$x^2 = \frac{e^9}{4}$

$x = \pm\frac{e^{4.5}}{2}$

The negative result fails to check.

$x = \frac{e^{4.5}}{2}$

35. $\ln(x^2 + 12) = \ln x + \ln 8$

$\ln(x^2 + 12) = \ln 8x$

$x^2 + 12 = 8x$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = \{2,6\}$

36. $\ln x + \ln(x+4) = \ln 5$

$\ln(x^2 + 4x) = \ln 5$

$x^2 + 4x = 5$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = \{-5,1\}$

-5 fails to check.

$x = 1$

37. $e^{\ln x} = 4$

$x = 4$

38. $200e^{-4x} = 15$

$e^{-4x} = \frac{3}{40}$

$-4x = \ln\left(\frac{3}{40}\right)$

$x = -\frac{1}{4}\ln\left(\frac{3}{40}\right)$

39. $\ln x^2 = 10$

$x^2 = e^{10}$

$x = \pm e^5$

40. $9 - 2e^x = 7$

$-2e^x = -2$

$e^x = 1$

$x = 0$

41. $\ln\sqrt{x+2} = 1$

$\sqrt{x+2} = e$

$x + 2 = e^2$

$x = e^2 - 2$

42. $\ln(x-2)^2 = 12$

$(x-2)^2 = e^{12}$

$x - 2 = \pm e^6$

$x = 2 \pm e^6$